# Uncertainty calculation for AMN impedance contribution using the Monte Carlo Method

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***Abstract*—The probability density function (PDF) of the error is derived for several frequencies, taking different source impedances of the EUT into consideration. The assumption of a triangular distribution cannot be confirmed. It is shown that the coverage interval is not always correlated to the minima and maxima of the PDF, as it is for the worst case analysis. The phase requirement for the AMN impedance is questionable, since the probability for the circumstance where a problem really occurs is neglectable. Further efforts must be put in the improvement of the measurement model, since the results heavily depend on the assumption of the EUT source impedance.**

***Keywords—artifical mains network, disturbance voltage measurement, impedance tolerance, measurement uncertainty, component; monte carlo method***

## I. Introduction

The generic standard and several product standard of CISPR require conducted disturbance voltage measurements. They refer to CISPR 16-2-1 [1], where the measurement method is defined. The equipment under test (EUT) is placed at a distance of 40 cm in front of a metallic wall on a wooden table. At a distance of 80 cm the artificial mains V-network (V-AMN) is placed and connected to the EUT via its meandered mains cable. The V-AMN has three purposes; first is to terminate the EUT cable with a defined RF impedance with respect to ground, the second is to conduct the voltage at the terminating impedance to an EMI receiver, the third is to isolate EUT mains from laboratory mains to avoid false measurement due to ambient voltage on laboratory mains. The specifications of the V-AMN are given in CISPR 16-1-2 [2]. There are two type of V-AMNs defined to cover the frequency range from 9 kHz to 30 MHz, the 50 Ω/50 µH + 5 Ω V-AMN from 9 kHz to 150 kHz and the 50 Ω/50 µH V-AMN from 150 kHz to 30 MHz. In practice this differentiation is irrelevant since commercial available V-AMN fulfil the impedance requirements of both ranges.

It is state-of-the-art to have guidance for measurement uncertainty calculation for each measurement method within an international standard. In the technical report CISPR/TR 16-4-1 [3] background information about the measurement principle and an uncertainty model is given. This model is explained in section II.

A measurement uncertainty calculation example budget is found in CISPR 16-4-2 [4]. The dominating contributors are the uncertainty of the EMI receiver and the tolerance of the impedance of the V-AMN. This paper investigates this influence quantity by using numerical methods.

## II. Measurement Model

In CISPR/TR 16-4-1 a measurement model for the conducted disturbance measurement method is given. The disturbance source is modelled by the disturbance voltage $V_d$ and the source impedance $Z_d$. In case of an ideal setup the voltage $V_{nom}$ is measured at the nominal impedance of the V-AMN $Z_{nom}$, see Fig. 1(a). Due to tolerances the real impedance will deviate from the nominal impedance. In this case the source is terminated with the impedance $Z_{AMN}$, where the disturbance voltage $V_{AMN}$ is measured, see Fig. 1(b).

The measurement error is defined in a log scale, see (1).

$$\Delta V = 20\log\left(\left|\frac{V_{AMN}}{V_{nom}}\right|\right) \tag{1}$$

In CISPR 16-1-2 the tolerance of the impedance is specified. Boundaries are given for the magnitude with ± 20 % and for the phase with ±11.5° across the whole frequency band. This means the tolerance is an annulus sector around the nominal impedance, see Fig. 2. Both boundaries are not independent from each other. The phase tolerance is calculated from the magnitude tolerance, see (17).

CISPR 16-1-2 is the only EMC standard, where a phase tolerance is specified.

There is an ambiguity between the specification in CISPR 16-1-2 and the impact of the tolerance given in CISPR 16-4-2. While a tolerance sector is defined, the calculation of the impact is based on a tolerance circle.

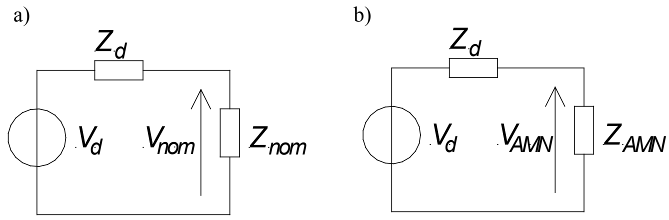



Fig. 1. Disturbance voltage measurement model a) ideal case b) real case, see [3]

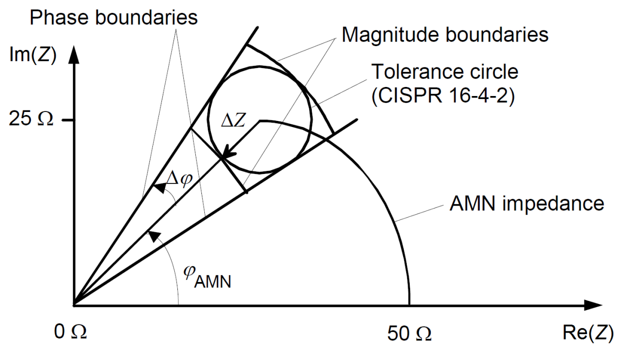



Fig. 2. AMN impedance tolerance of CISPR 16-1-2

## III. Worst Case Analysis

In CISPR 16-4-2 a worst case analysis is performed to get the measurement error. The calculation is based on the work of Stecher [5], which uses reflection coefficients normalized to system impedance $Z_0$=50 **Ω**, see (2) to (4)

$$\Gamma_{nom} = \frac{Z_{nom} - Z_0}{Z_{nom} + Z_0} \tag{2}$$

$$\Gamma_{AMN} = \frac{Z_{AMN} - Z_0}{Z_{AMN} + Z_0} \tag{3}$$

$$\Gamma_d = \frac{Z_d - Z_0}{Z_d + Z_0} \tag{4}$$

The voltages are derived by using (2) to (4) and a voltage divider, see (5) and (6)

$$V_{nom} = \frac{Z_{nom}}{Z_d + Z_{nom}} V_d = \frac{(1+\Gamma_{nom})(1-\Gamma_d)}{2(1-\Gamma_{nom}\Gamma_d)} V_d \tag{5}$$

$$V_{AMN} = \frac{Z_{AMN}}{Z_d + Z_{AMN}} V_d = \frac{(1+\Gamma_{AMN})(1-\Gamma_d)}{2(1-\Gamma_{AMN}\Gamma_d)} V_d \tag{6}$$

The measurement error is calculated using formula (1) to (6), see (7)

$$\Delta U = 20\log\left(\left|\frac{1+\Gamma_{AMN}}{1-\Gamma_{AMN}\Gamma_d}\,\frac{1-\Gamma_{nom}\Gamma_d}{1+\Gamma_{nom}}\right|\right) \tag{7}$$

Due to the impedance specification, $Z_{AMN}$ with a tolerance circle is calculated by (8). It is assumed that the worst case will occur if $Z_{AMN}$ is exactly on this circle and not inside the circle.

$$Z_{AMN} = Z_{nom} + 0.2\left|Z_{nom}\right| e^{j\Theta} \tag{8}$$

$$0 \le \Theta < 2\pi \tag{9}$$

For the source impedance is assumed that $|\Gamma_d|$=1. As before it is assumed that worst case will occur exactly on this circle and not inside the circle.

This calculation leads to a minimum/maximum of -3.07 dB/+3.60 dB at 53 kHz for the frequency range 9 kHz to 150 kHz. At 150 kHz values of -2.60 dB/+2.68 dB for the frequency range from 150 kHz to 30 MHz are found. These results are nearly identical with [5],[6] and [7] but have the opposite sign compared to the results given in CISPR 16-4-2. So a typo in the standard may be considered as reason for this discrepancy.

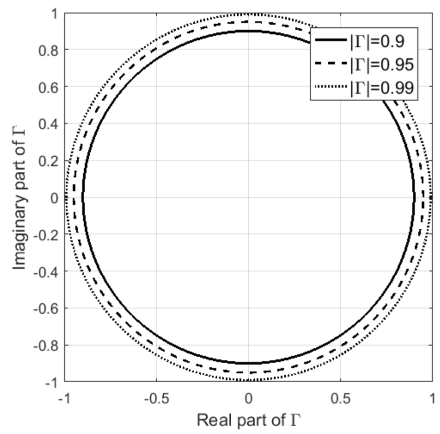
Due to the requirement of having a probability density function (PDF) for the measurement uncertainty calculation, a triangular distribution is assumed in CISPR 16-4-2. This triangular distribution should be asymmetric with the calculated minima and maxima as boundaries and an estimate of 0 dB. This assumption is based on the mentality that it is more likely to have values near the central than having values close to the extremes.

Equation (4) gives the transformation from the $Z$ domain to the Γ domain given. The inverse transformation can be easily derived, see (10)

$$Z_d = Z_0 \frac{1+\Gamma_d}{1-\Gamma_d} \tag{10}$$

As mentioned in section III, it is assumed that $|\Gamma_d|$=1. As consequence the real values of the source impedance $Z_d$ is zero. This fact can be seen if a circle in the Γ domain is transformed into the $Z$ domain, see Figure 3. If the radius approaches 1 the ellipse in the $Z$ domain will become a straight line on the imaginary axis.

a)




b)

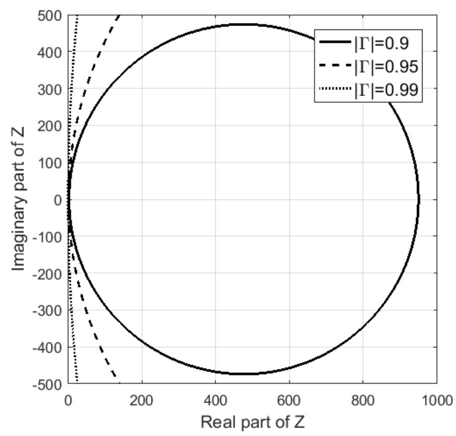



Fig. 3. Transformation of circles $|\Gamma_d|$=0.9, $|\Gamma_d|$=0.95 and $|\Gamma_d|$=0.99 a) Γ domain b) $Z$ domain

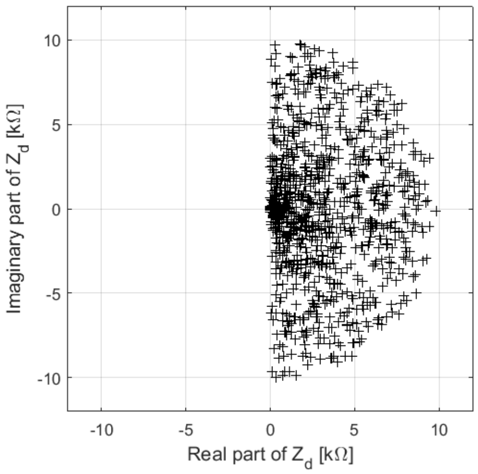



Fig. 4. MC sample for impedance $Z_d$, $Z_{max}$=10 kΩ, 1000 MC points

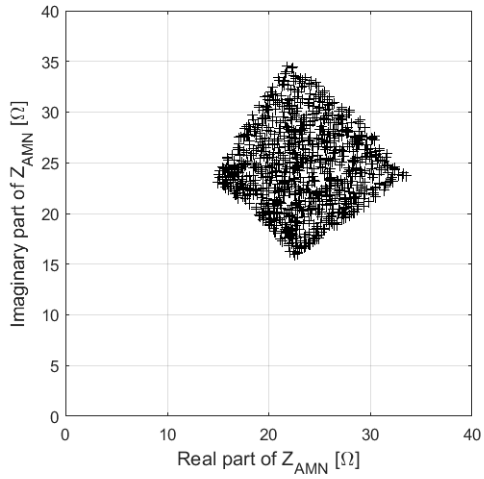



Fig. 5. MC sample for impedance $Z_{AMN}$, 150 kHz, 1000 MC points

From this assumption it is obvious why it was necessary to have a phase tolerance of the AMN impedance. Without a phase tolerance the AMN impedance could be pure imaginary too. So the voltage divider, see Fig. 1, could become a series LC resonator and the measurement error infinite.

A triangular distribution is described by three parameters, the lower limit *a*, the peak location *c* and the upper limit *b* [8]. The expected value is calculated by the mean value of *a*, *b* and *c*. If we force that the expected value and the peak value *c* is 0 dB, we will get *a*=-*b*. So a triangular distribution with these assumptions cannot be asymmetrical.

## IV. Statistical Analysis

The worst case analysis as a method to calculate measurement uncertainty was outdated by the introduction of statistical methods more than 25 years ago. Also Carrobi [7] suggested that a "probabilistic" approach is more suitable.

The Monte Carlo (MC) method has several advantages for measurement uncertainty calculation problems [10]. A closed solution is not required anymore and dealing with complex numbers is simplified. It is easy to implement it with modern math software and the execution times are reasonable. The bases of the MC method are samples that are filled with random numbers according to the assumed PDF. Each sample represents a statistical variable, e.g. the impedance $Z_{AMN}$, and is used to calculate the output variable, $\Delta V$ in this case. The measurement uncertainty is derived directly from the PDF of $\Delta V$ by calculating the 95 % intervals.

Also for the statistical analysis it is required to make some assumptions. With the MC method it is possible to assume the source impedance $Z_d$ directly, see (11) to (13)

$$Z_d = Ze^{j\Theta} \tag{11}$$

$$0 \le Z \le Z_{\max} \tag{12}$$

$$-\frac{\pi}{2} \le \Theta \le +\frac{\pi}{2} \tag{13}$$

The absolute value $Z$ and phase $\Theta$ are assumed to be uniform distributed. Due to physics the phase is bounded because negative real values have to be omitted. An example of this limited half space is given in Figure 4.

On the contrary to the worst case analysis it is possible to assume the AMN impedance correctly as defined by the standard. Instead of the tolerance circle an annulus sector is assumed, which is described by (14) to (16). In Fig. 5 it can be clearly seen that also values inside the sector are possible and not only on its boundary.

$$Z_{AMN} = \left(\left|Z_{nom}\right|\left(1+\alpha\right)\right)e^{j\left(\arg Z_{nom}+\Theta\right)} \tag{14}$$

$$-0.2 \le \alpha \le 0.2 \tag{15}$$

$$-\varphi \le \Theta \le \varphi \tag{16}$$

$$\varphi = \sin^{-1}\left(0.2\right) = 11.5° \tag{17}$$

Equation (17) shows the derivation of the phase limit from the magnitude limit.

### A. Simulation result

The goal was to derive the relationship of the PDF shape and the measurement error from the maximal source impedance $Z_{max}$ and the frequency. To do so $Z_{max}$ was varied between 1 Ω and 10 kΩ and the measurement error is calculated at 9 kHz, 150 kHz and 30 MHz, see Fig. 6. It can be seen that the measurement error increases with $Z_{max}$, but saturates at a certain value. For a low value of 1 Ω the PDF has a peak at 0 dB and fall off at the side very quickly, see Fig. 6(a)(d)(g). The frequency dependency of the measurement error can be explained by the frequency dependency of the AMN impedance. For a high value of 10 kΩ the PDF is similar to a uniform distribution, but with a "lean-to roof", see Fig. 6(c)(f)(i). Also here a frequency dependency can be recognized on the sharper and smoother edges of the PDF.

The results for a value of 100 Ω, see Fig. 6(b)(e)(h), can be seen as transition between the cases at 9 kHz and 30 MHz. At a frequency of 9 kHz, see Fig. 6(b), the shape of the 10 kΩ

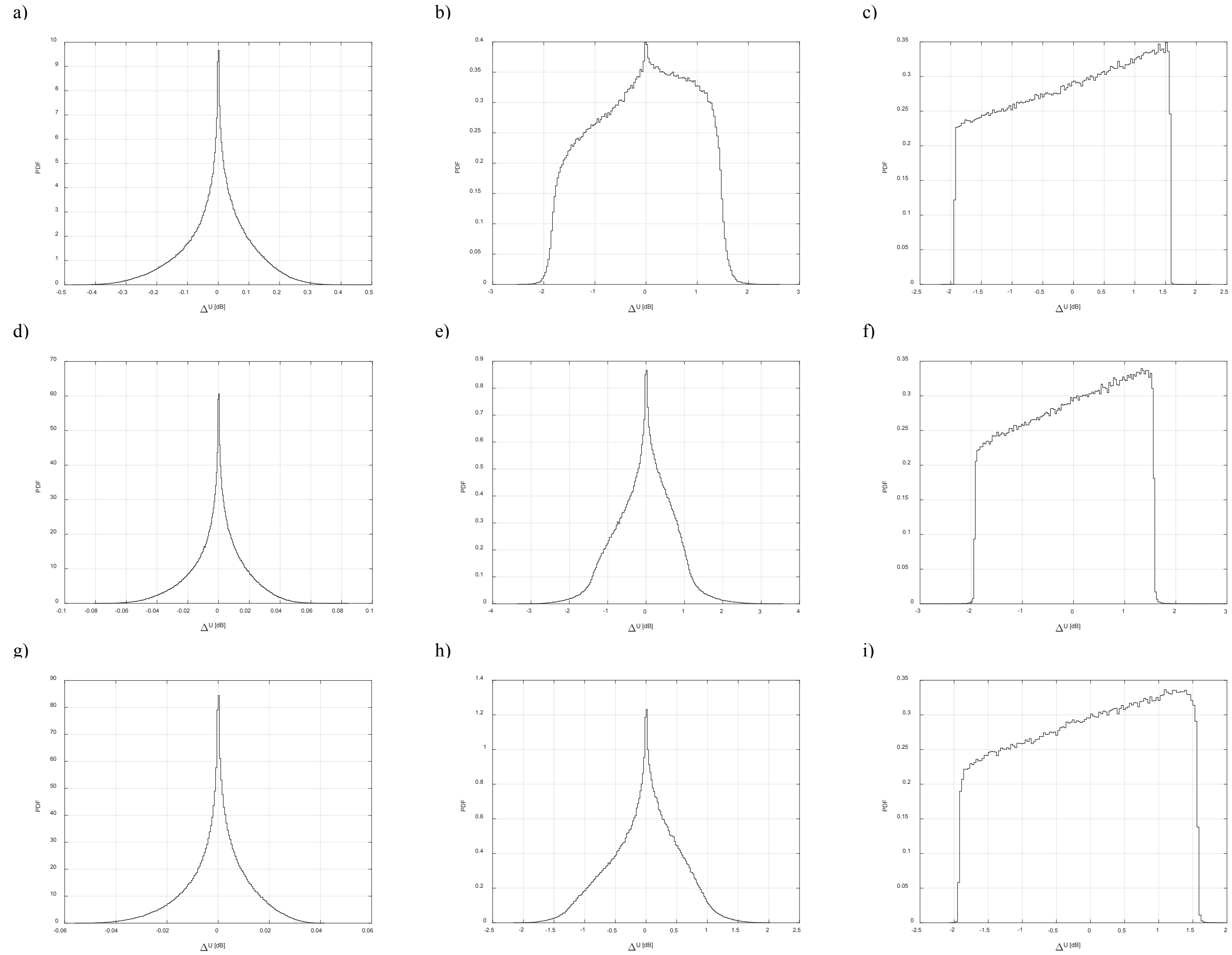



Fig. 6. Propability density function of the measurement error at 9 kHz a) $Z_{max}$=1 Ω b) $Z_{max}$=100 Ω c) $Z_{max}$=10 kΩ, at 150 kHz d) $Z_{max}$=1 Ω e) $Z_{max}$=100 Ω f) $Z_{max}$=10 kΩ and at 30 MHz g) $Z_{max}$=1 Ω h) $Z_{max}$=100 Ω i) $Z_{max}$=10 kΩ

can be seen, but with smoother edges. The 0 dB peak can be recognized.

At 150 kHz and 30 MHz the results look more similar to the result at 1 Ω. The fall off at the side is not that steep, but the PDF is dominated by the peak at 0 dB. With some fantasy, the PDF can be identified as distorted asymmetric triangular distribution, especially for the 30 MHz result, see Fig. 6(h).

From these PDFs and results at 10 Ω and 100 Ω, the uncertainty for a level of confidence of 95 % is calculated, see Table I.

TABLE I. UNCERTAINTY (95%) FOR SEVERAL VALUES OF $Z_{MAX}$

| **Frequency** | ***Zmax*** | | | | |
|---|---|---|---|---|---|
| | ***1 Ω*** | ***10 Ω*** | ***100 Ω*** | ***1 kΩ*** | ***10 kΩ*** |
| 9 kHz | -0.24dB/ +0.21dB | -1.20dB/ +1.04dB | -1.72dB/ +1.43dB | -1.81dB/ +1.49dB | -1.83dB/ +1.51dB |
| 150 kHz | -0.04dB/ +0.03dB | -0.35dB/ +0.32dB | -1.49dB/ +1.35dB | -1.77dB/ +1.47dB | -1.82dB/ +1.50dB |
| 30 MHz | -0.03dB/ +0.02dB | -0.25dB/ +0.21dB | -1.17dB/ +1.00dB | -1.71dB/ +1.41dB | -1.81dB/ +1.49dB |

Since the PDFs are not Gaussian the intervals cannot be calculated by multiplying the standard deviation by the coverage factor. Therefore the cumulative distribution function (CDF) is derived from the PDF. From the CDF the interval is determined from the 2.5 % and the 97.5 % point.

### B. *Discussion*

The results of section IV(A) can be explained as following: If the source impedance is much lower than the terminating impedance it does not have an influence to the measured voltage. If the source impedance is much higher than the terminating impedance the current through the voltage divider is determined by the source impedance.

The measured voltage, proportional to the current, depends on the tolerance of the terminating impedance. The minimum and maximum values of $\Delta V$ can be estimated, see (18).

$$\Delta V = 20\log\left(1 \pm 0.2\right) = -1.94dB / +1.58dB \qquad (18)$$

These numbers fit well to the PDFs given in Fig. 6(c)(f)(i). The results given in Table I are slightly smaller, because they are valid for 95 % confidence and are not minimum and maximum values.

The comparison of the results of the statistical analysis and the worst case analysis is difficult, because several statistical parameter of the PDF of $\Delta V$ need to be taken into consideration. If the minima and maxima at a frequency of 150 kHz are compared only marginal differences can be observed. The PDF, shown in Fig. 6(e), gives-2.7 dB/+2.8 dB; [4] gives -2.7 dB/+2.6 dB and [7] gives-2.5 dB/+2.6 dB. If the standard deviations are compared, also small deviation can be seen. The calculation leads to 1.01 dB, respectively 1.08 dB and 1.04 dB. For the coverage interval of 95 % large differences are observed, due to the different shapes of the PDF. As seen in Table I, values of -1.82 dB/+1.50 dB are calculated for the statistical analysis. The resulting PDF of a calculation using the Gaussian error propagation is always a normal distribution, due to the assumption of the central limit theorem. Even if the AMN impedance tolerance is the dominating contribution in the measurement uncertainty budget, the normal distribution is assumed instead of the triangular distribution. So the coverage interval for 95 % coverage is calculated by multiplying the standard deviation by the coverage factor $k$=2. With the values of [4], this leads to the symmetric interval of -2.16 dB/+2.16 dB.

As seen in Table I, only a neglectable frequency dependency can be observed for the coverage interval. For the minima and maxima a strong frequency dependency is seen [4][7].

### C. Consequences for AMN phase limit

From a statistical point of view, the results of section III should be reconsidered, because there is a very low probability that the worst case will take effect.

Instead of a phase limit given in formula (14) to (16), no limit is given in formula (19) to (21). However the phase is limited to avoid a negative real part.

$$Z_{AMN} = \left(\left|Z_{nom}\right|\left(1+\alpha\right)\right)e^{j\Theta} \quad (19)$$

$$-0.2 \le \alpha \le 0.2 \quad (20)$$

$$-\frac{\pi}{2} \le \Theta \le \frac{\pi}{2} \quad (21)$$

The result of this simulation is shown in Table 2. For very small values of 1 Ω the influence is larger but still acceptable. For values between 10 Ω and 1 kΩ the error can reach values

TABLE II. UNCERTAINTY (95%) FOR SEVERAL VALUES OF $Z_{MAX}$, NO PHASE LIMIT

| Frequency | $Z_{max}$ | | | | |
|---|---|---|---|---|---|
| | **1 Ω** | **10 Ω** | **100 Ω** | **1 kΩ** | **10 kΩ** |
| 9 kHz | -1.04dB/ +1.57dB | -3.54dB/ +7.46dB | -2.15dB/ +3.18dB | -1.82dB/ +1.57dB | -1.83dB/ +1.52dB |
| 150 kHz | -0.24dB/ +0.27dB | -2.16dB/ +2.46dB | -5.22dB/ +6.68dB | -2.44dB/ +2.54dB | -1.85dB/ +1.54dB |
| 30 MHz | -0.09dB/ +0.16dB | -0.74dB/ +1.60dB | -2.22dB/ +7.46dB | -1.84dB/ +3.17dB | -1.80dB/ +1.58dB |

up to 7.46 dB. For very high values of 10 kΩ only a neglectable influence is seen, because the probability to find a resonance is very low. If the assumption of an unlimited source impedance is still followed, the phase limit should be removed.

## V. CONCLUSION

In this paper statistical methods had been used to calculate the influence of the actual impedance of an AMN to the voltage measurement. To do this known models and assumptions had been used. It was possible to derive the PDF of the measurement error directly.

A good agreement to previous papers is found for certain parameters like minima/maxima and standard deviation. As Carobbi [7] already mentioned, the two different perspectives worst case and statistic analysis lead to different results.

The requirement of an AMN impedance phase limit cannot be confirmed, if an unlimited source impedance is assumed. The chance that a resonance really occurs is marginal.

In the literature always an unlimited source impedance is assumed, due to limited knowledge about the behaviour of a real EUT. Since this has a major influence on the result, further work will be an examination. Depending on the disturbance type, common mode or differential mode, the test setup itself will influence the source impedance due to the capacitive coupling of the EUT to ground and the inductance of the mains cable. Typically an EMC filter is used in the EUT to reduce the disturbance voltage of the power supply. The input impedance of this filter could be used as a first step for further work.

## REFERENCES


[1] CISPR 16-2-1:2014 +AMD1:2017 “Specification for radio disturbance and immunity measuring apparatus and methods - Part 2-1: Methods of measurement of disturbances and immunity - Conducted disturbance measurements”, Edition 3.1, 2017-06-30

[2] CISPR 16-1-2:2014+AMD1:2017 „Specification for radio disturbance and immunity measuring apparatus and methods - Part 1-2: Radio disturbance and immunity measuring apparatus - Coupling devices for conducted disturbance measurements”, Edition 2.1, 2017-11-07

[3] CISPR TR 16-4-1:2009 „Specification for radio disturbance and immunity measuring apparatus and methods - Part 4-1: Uncertainties, statistics and limit modelling - Uncertainties in standardized EMC tests“, Edition 2.0, 2009-02-23

[4] CISPR 16-4-2:2011+AMD1:2014+AMD2:2018 “Specification for radio disturbance and immunity measuring apparatus and methods - Part 4-2: Uncertainties, statistics and limit modelling - Measurement instrumentation uncertainty”, Edition 2.2, 2018-08-15

[5] Manfred Stecher “Uncertainty in RF Disturbance Measurements: Revision of CISPR 16-4-2” EMC’09/Kyoto, 23R1-1

[6] Rohde& Schwarz “Impedance Uncertainty Contribution of Artifical Networks (AN, AMN and ISN)”, Application note 1EE23, 2010-09

[7] Carlo F. M. Carobbi, Manfred Stecher “The Effect of the Imperfect Realization of the Artificial Mains Network Impedance on the Reproducibility of Conducted Emission Measurements”, IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY, VOL. 54, NO. 5, OCTOBER 2012

[8] Weisstein, Eric W. "Triangular Distribution." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/TriangularDistribution.html

[9] ISO/IEC Guide 98:1993 “Guide to the expression of uncertainty in measurement (GUM)”, 1993-01, revised by ISO/IEC Guide 98:2008